Integration

Question Paper 4

Level	A Level
Subject	Mathematics (Pure)
Exam Board	AQA
Module	Core 2
Topic	Calculus
Sub Topic	Integration
Booklet	Question Paper 4

Time Allowed: **84 minutes**

Score: **/69**

Percentage: **/100**

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

1 (a) The expression $\left(1+\frac{4}{x^2}\right)^3$ can be written in the form

$$1+\frac{p}{x^2}+\frac{q}{x^4}+\frac{64}{x^6}$$

By using the binomial expansion, or otherwise, find the values of the integers p and q.

(3)

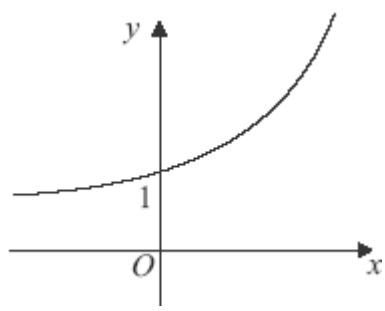
(b) (i) Hence find $\int\left(1+\frac{4}{x^2}\right)^3 \mathrm{d}x$.

(4)

(ii) Hence find the value of $\int_1^2\left(1+\frac{4}{x^2}\right)^3 \mathrm{d}x$.

(2)
(Total 9 marks)

2 The diagram shows a sketch of the curve with equation $y = 6^x$.



(a) (i) Use the trapezium rule with five ordinates (four strips) to find an approximate value for $\int_0^2 6^x \mathrm{d}x$, giving your answer to three significant figures.

(4)

(ii) Explain, with the aid of a diagram, whether your approximate value will be an overestimate or an underestimate of the true value of $\int_0^2 6^x \mathrm{d}x$.

(2)

(b) (i) Describe a single geometrical transformation that maps the graph of $y = 6^x$ onto the graph of $y = 6^{3x}$.

(2)

(ii) The line $y = 84$ intersects the curve $y = 6^{3x}$ at the point A. By using logarithms, find the x-coordinate of A, giving your answer to three decimal places.

(4)

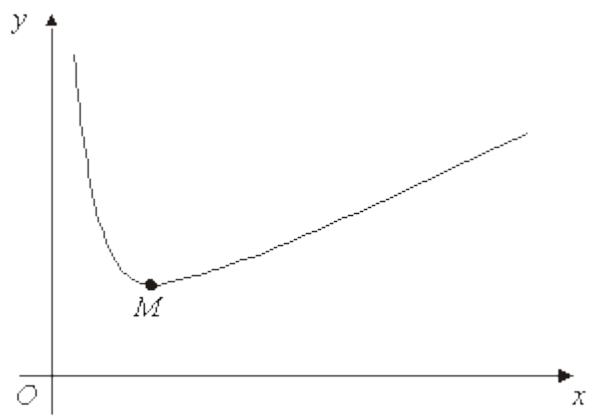
(c) The graph of $y = 6^x$ is translated by $\begin{bmatrix} 1 \\ -2 \end{bmatrix}$ to give the graph of the curve with equation $y = \mathrm{f}(x)$. Write down an expression for $\mathrm{f}(x)$.

(2)
(Total 14 marks)

3 Use the trapezium rule with four ordinates (three strips) to find an approximate value for $\int_0^3 \sqrt{2^x}\,\mathrm{d}x$ giving your answer to three decimal places.

(Total 4 marks)

4 A curve C is defined for $x > 0$ by the equation $y = x + 1 + \frac{4}{x^2}$ and is sketched below.



(a) (i) Given that $y = x + 1 + \frac{4}{x^2}$, find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

(3)

(ii) The curve C has a minimum point M. Find the coordinates of M.

(4)

(iii) Find an equation of the normal to C at the point (1, 6).

(4)

(b) (i) Find $\int \left(x + 1 + \frac{4}{x^2} \right) \mathrm{d}x$.

(3)

(ii) Hence find the area of the region bounded by the curve C, the lines $x = 1$ and $x = 4$ and the x-axis.

(2)
(Total 16 marks)

5 (a) Simplify:

(i) $x^{\frac{3}{2}} \times x^{\frac{1}{2}}$;

(1)

(ii) $x^{\frac{3}{2}} \div x$;

(1)

(iii) $\left(x^{\frac{3}{2}} \right)^2$

(1)

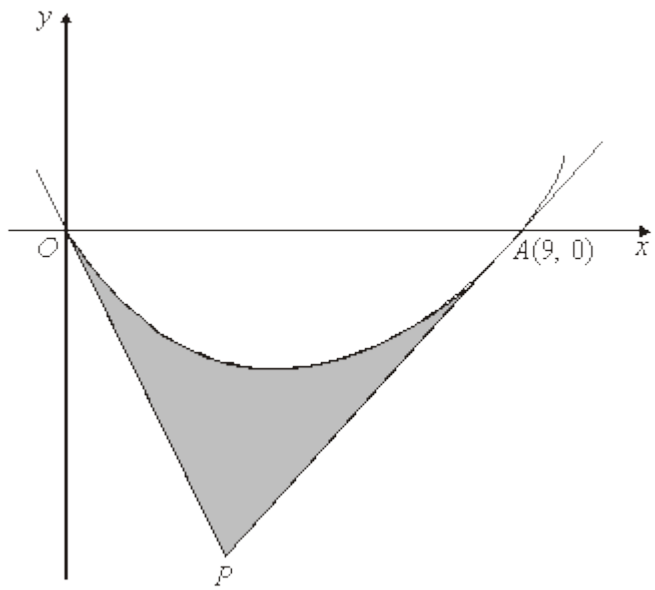
(b) (i) Find $\int 3x^{\frac{1}{2}} \mathrm{d}x$.

(3)

(ii) Hence find the value of $\int_1^9 3x^{\frac{1}{2}} \mathrm{d}x$.

(2)
(Total 8 marks)

6 A curve, drawn from the origin O, crosses the x-axis at the point $A(9, 0)$. Tangents to the curve at O and A meet at the point P, as shown in the diagram.



The curve, defined for $x \geq 0$, has equation

$$y = x^{\frac{3}{2}} - 3x$$

(a) Find $\frac{dy}{dx}$.

(2)

(b) (i) Find the value of $\frac{dy}{dx}$ at the point O and hence write down an equation of the tangent at O.

(2)

(ii) Show that the equation of the tangent at $A(9, 0)$ is $2y = 3x - 27$.

(3)

(iii) Hence find the coordinates of the point P where the two tangents meet.

(3)

(c) Find $\int \left(x^{\frac{3}{2}} - 3x \right) dx$.

(3)

(d) Calculate the area of the shaded region bounded by the curve and the tangents OP and AP.

(5)
(Total 18 marks)