Mark Scheme 4753
January 2007

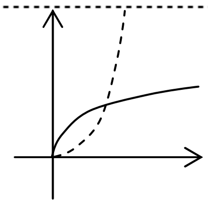
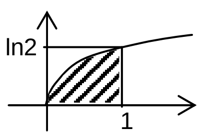
Section A

1 (i) P is (2, 1)	B1	
(ii) $\|x\|=1\frac{1}{2}$ $\Rightarrow x=(-1\frac{1}{2})\text{ or }1\frac{1}{2}$ $\|x-2\|+1=1\frac{1}{2}\Rightarrow\|x-2\|=\frac{1}{2}$ $\Rightarrow x=(2\frac{1}{2})\text{ or }1\frac{1}{2}$	M1 A1 M1 E1	allow x = 1½ unsupported or $\left\|1\frac{1}{2}-2\right\|+1=\frac{1}{2}+1=1\frac{1}{2}$
or by solving equation directly: $\|x-2\|+1=\|x\|$ $\Rightarrow$ $2-x+1=x$ $\Rightarrow$ x = 1½ $\Rightarrow$ $y = \|x\| = 1½$	M1 M1 A1 E1 [4]	equating from graph or listing possible cases
2 $\int_1^2 x^2\ln x\,dx$ $u=\ln x$ $dv/dx=x^2\Rightarrow v=\frac{1}{3}x^3$ $=\left[\frac{1}{3}x^3\ln x\right]_1^2-\int_1^2\frac{1}{3}x^3.\frac{1}{x}dx$ $=\frac{8}{3}\ln 2-\int_1^2\frac{1}{3}x^2dx$ $=\frac{8}{3}\ln 2-\left[\frac{1}{9}x^3\right]_1^2$ $=\frac{8}{3}\ln 2-\frac{8}{9}+\frac{1}{9}$ $=\frac{8}{3}\ln 2-\frac{7}{9}$	M1 A1 A1 M1 A1 cao [5]	Parts with $u=\ln x$ $dv/dx=x^2\Rightarrow v=x^3/3$ $\left[\frac{1}{9}x^3\right]$ substituting limits o.e. – not ln 1
3 (i) When t = 0, V = 10 000 $\Rightarrow$ 10 000 = $Ae^0 = A$ When t = 3, V = 6000 $\Rightarrow$ 6000 = 10 000 e^{-3k} $\Rightarrow$ $-3k$ = ln(0.6) = –0.5108… $\Rightarrow$ k = 0.17(02…)	M1 A1 M1 M1 A1 [5]	10 000 = Ae^0 A = 10 000 taking lns (correctly) on their exponential equation - not logs unless to base 10 art 0.17 or –(ln 0.6)/3 oe
(ii) 2000 = 10 000e^{-kt} $\Rightarrow$ $-kt$ = ln 0.2 $\Rightarrow$ t = –ln 0.2 / k = 9.45 (years)	M1 A1 [2]	taking lns on correct equation (consistent with their k) allow art 9.5, but not 9.

4 Perfect squares are 0, 1, 4, 9, 16, 25, 36, 49, 64, 81 none of which end in a 2, 3, 7 or 8. Generalisation: no perfect squares end in a 2, 3, 7 or 8.	M1 E1 B1 [3]	Listing all 1- and 2- digit squares. Condone absence of 0^2, and listing squares of 2 digit nos (i.e. $0^2 – 19^2$) For extending result to include further square numbers.
5 (i) $y=\frac{x^2}{2x+1}$ $\Rightarrow \frac{dy}{dx}=\frac{(2x+1)2x-x^2.2}{(2x+1)^2}$ $=\frac{2x^2+2x}{(2x+1)^2}=\frac{2x(x+1)}{(2x+1)^2}$ *	M1 A1 A1 E1 [4]	Use of quotient rule (or product rule) Correct numerator – condone missing bracket provided it is treated as present Correct denominator www –do not condone missing brackets
(ii) $\frac{dy}{dx}=0$ when $2x(x + 1) = 0$ $\Rightarrow$ $x = 0$ or -1 $y = 0$ or -1	B1 B1 B1 B1 [4]	Must be from correct working: SC −1 if denominator = 0
6(i) QA = 3 – *y*, PA = 6 – (3 – *y*) = 3 + *y* By Pythagoras, $PA^2 = OP^2 + OA^2$ $\Rightarrow$ $(3 + y)^2 = x^2 + 3^2 = x^2 + 9$. *	B1 B1 E1 [3]	must show some working to indicate Pythagoras (e.g. $x^2 + 3^2$)
(ii) Differentiating implicitly: $2(y+3)\frac{dy}{dx}=2x$ $\Rightarrow \frac{dy}{dx}=\frac{x}{y+3}$ *	M1 E1	Allow errors in RHS derivative (but not LHS) - notation should be correct brackets must be used
or $9 + 6y + y^2 = x^2 + 9$ $\Rightarrow 6y + y^2 = x^2$ $\Rightarrow (6+2y)\frac{dy}{dx}=2x$ $\Rightarrow \frac{dy}{dx}=\frac{x}{y+3}$	M1 E1	Allow errors in RHS derivative (but not LHS) - notation should be correct brackets must be used
or $y = \sqrt{(x^2+ 9)} – 3 \Rightarrow dy/dx = ½(x^2+9)^{-1/2}.2x$ $=\frac{x}{\sqrt{x^2+9}}=\frac{x}{y+3}$	M1 E1	(cao)
(iii) $\frac{dy}{dt}=\frac{dy}{dx}.\frac{dx}{dt}$ $=\frac{4}{2+3}\times 2$ $=\frac{8}{5}$	M1 A1 A1 [3]	chain rule (soi)

Section B

7(i) When $x = -1$, $y = -1\sqrt{0} = 0$ Domain $x \ge -1$	E1 B1 [2]	Not $y \ge -1$
(ii) $\frac{dy}{dx} = x.\frac{1}{2}(1+x)^{-1/2} + (1+x)^{1/2}$ $= \frac{1}{2}(1+x)^{-1/2}[x + 2(1+x)]$ $= \frac{2+3x}{2\sqrt{1+x}}$ *	B1 B1 M1 E1	$x.\frac{1}{2}(1+x)^{-1/2}$ … + $(1+x)^{1/2}$ taking out common factor or common denominator www
or $u = x + 1 \Rightarrow du/dx = 1$ $\Rightarrow y = (u-1)u^{1/2} = u^{3/2} - u^{1/2}$ $\Rightarrow \frac{dy}{du} = \frac{3}{2}u^{\frac{1}{2}} - \frac{1}{2}u^{-\frac{1}{2}}$ $\Rightarrow \frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx} = \frac{3}{2}(x+1)^{\frac{1}{2}} - \frac{1}{2}(x+1)^{-\frac{1}{2}}$ $= \frac{1}{2}(x+1)^{-\frac{1}{2}}(3x+3-1)$ $= \frac{2+3x}{2\sqrt{1+x}}$ *	M1 A1 M1 E1 [4]	taking out common factor or common denominator
(iii) $dy/dx = 0$ when $3x + 2 = 0$ $\Rightarrow x = -2/3$ $\Rightarrow y = -\frac{2}{3}\sqrt{\frac{1}{3}}$ Range is $y \ge -\frac{2}{3}\sqrt{\frac{1}{3}}$	M1 A1cao A1 B1 ft [4]	o.e. not $x \ge -\frac{2}{3}\sqrt{\frac{1}{3}}$ (ft their y value, even if approximate)
(iv) $\int_{-1}^{0} x\sqrt{1+x}\,dx$ let $u = 1 + x$, $du/dx = 1 \Rightarrow du = dx$ when $x = -1$, $u = 0$, when $x = 0$, $u = 1$ $= \int_0^1 (u-1)\sqrt{u}\,du$ $= \int_0^1 (u^{3/2} - u^{1/2})\,du$ *	M1 B1 M1 E1	$du = dx$ or $du/dx = 1$ or $dx/du = 1$ changing limits – allow with no working shown provided limits are present and consistent with dx and du. $(u-1)\sqrt{u}$ www – condone only final brackets missing, otherwise notation must be correct
$= \left[\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}\right]_0^1$ $= \pm\frac{4}{15}$	B1 B1 M1 A1cao [8]	$\frac{2}{5}u^{5/2}, -\frac{2}{3}u^{3/2}$ (oe) substituting correct limits (can imply the zero limit) $\pm\frac{4}{15}$ or ± 0.27 or better, not 0.26

8 (i) $f'(x) = 2(e^x - 1)e^x$ When $x = 0$, f'(0) = 0 When $x = \ln 2$, $f'(\ln 2) = 2(2 - 1)2$ $= 4$	M1 A1 B1dep M1 A1cao [5]	or $f(x) = e^{2x} - 2e^x + 1$ M1 (or $(e^x)^2 - 2e^x + 1$ plus correct deriv of $(e^x)^2$) $\Rightarrow f'(x) = 2e^{2x} - 2e^x$ A1 derivative must be correct, www $e^{\ln 2} = 2$ soi
(ii) $y = (e^x - 1)^2$ $x \leftrightarrow y$ $x = (e^y - 1)^2$ $\Rightarrow$ $\sqrt{x} = e^y - 1$ $\Rightarrow$ $1 + \sqrt{x} = e^y$ $\Rightarrow$ $y = \ln(1 + \sqrt{x})$	M1 M1 E1	reasonable attempt to invert formula taking lns similar scheme of inverting $y = \ln(1 + \sqrt{x})$
or $gf(x) = g((e^x - 1)^2)$ $= \ln(1 + e^x - 1)$ $= x$	M1 M1 E1	constructing gf or fg $\ln(e^x) = x$ or $e^{\ln(1+\sqrt{x})} = 1 + \sqrt{x}$
 Gradient at (1, ln 2) = ¼	B1 B1ft [5]	reflection in $y = x$ (must have infinite gradient at origin)
(iii) $\int (e^x - 1)^2 dx = \int (e^{2x} - 2e^x + 1) dx$ $= \frac{1}{2}e^{2x} - 2e^x + x + c$ * $\int_0^{\ln 2} (e^x - 1)^2 dx = \left[\frac{1}{2}e^{2x} - 2e^x + x\right]_0^{\ln 2}$ $= \frac{1}{2}e^{2\ln 2} - 2e^{\ln 2} + \ln 2 - (\frac{1}{2} - 2)$ $= 2 - 4 + \ln 2 - \frac{1}{2} + 2$ $= \ln 2 - \frac{1}{2}$	M1 E1 M1 M1 A1 [5]	expanding brackets (condone e^{x^2}) substituting limits $e^{\ln 2} = 2$ used must be exact
(iv)  Area = $1 \times \ln 2 - (\ln 2 - \frac{1}{2})$ $= \frac{1}{2}$	M1 B1 A1cao [3]	subtracting area in (iii) from rectangle rectangle area = $1 \times \ln 2$ must be supported