

**ADVANCED GCE UNIT
MATHEMATICS (MEI)**

Methods for Advanced Mathematics (C3)

MONDAY 11 JUNE 2007

4753/01

Afternoon
Time: 1 hour 30 minutes

Additional materials:

Answer booklet (8 pages)

Graph paper

MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- The total number of marks for this paper is 72.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

Section A (36 marks)

1 (i) Differentiate $\sqrt{1+2x}$. [3]

(ii) Show that the derivative of $\ln(1 - e^{-x})$ is $\frac{1}{e^x - 1}$. [4]

2 Given that $f(x) = 1 - x$ and $g(x) = |x|$, write down the composite function $gf(x)$.

On separate diagrams, sketch the graphs of $y = f(x)$ and $y = gf(x)$. [3]

3 A curve has equation $2y^2 + y = 9x^2 + 1$.

(i) Find $\frac{dy}{dx}$ in terms of x and y . Hence find the gradient of the curve at the point A (1, 2). [4]

(ii) Find the coordinates of the points on the curve at which $\frac{dy}{dx} = 0$. [4]

4 A cup of water is cooling. Its initial temperature is 100°C . After 3 minutes, its temperature is 80°C .

(i) Given that $T = 25 + ae^{-kt}$, where T is the temperature in $^\circ\text{C}$, t is the time in minutes and a and k are constants, find the values of a and k . [5]

(ii) What is the temperature of the water

(A) after 5 minutes,

(B) in the long term? [3]

5 Prove that the following statement is false.

For all integers n greater than or equal to 1, $n^2 + 3n + 1$ is a prime number. [2]

- 6 Fig. 6 shows the curve $y = f(x)$, where $f(x) = \frac{1}{2} \arctan x$.

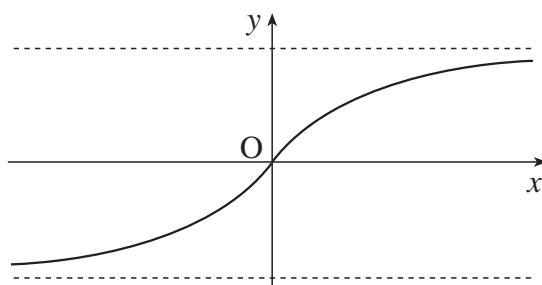


Fig. 6

- (i) Find the range of the function $f(x)$, giving your answer in terms of π . [2]
- (ii) Find the inverse function $f^{-1}(x)$. Find the gradient of the curve $y = f^{-1}(x)$ at the origin. [5]
- (iii) Hence write down the gradient of $y = \frac{1}{2} \arctan x$ at the origin. [1]

Section B (36 marks)

- 7 Fig. 7 shows the curve $y = \frac{x^2}{1 + 2x^3}$. It is undefined at $x = a$; the line $x = a$ is a vertical asymptote.

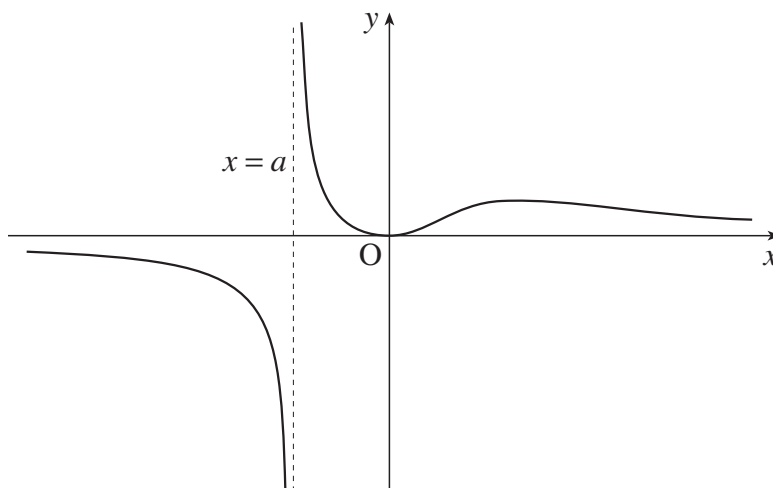


Fig. 7

- (i) Calculate the value of a , giving your answer correct to 3 significant figures. [3]
- (ii) Show that $\frac{dy}{dx} = \frac{2x - 2x^4}{(1 + 2x^3)^2}$. Hence determine the coordinates of the turning points of the curve. [8]
- (iii) Show that the area of the region between the curve and the x -axis from $x = 0$ to $x = 1$ is $\frac{1}{6} \ln 3$. [5]

- 8 Fig. 8 shows part of the curve $y = x \cos 2x$, together with a point P at which the curve crosses the x -axis.

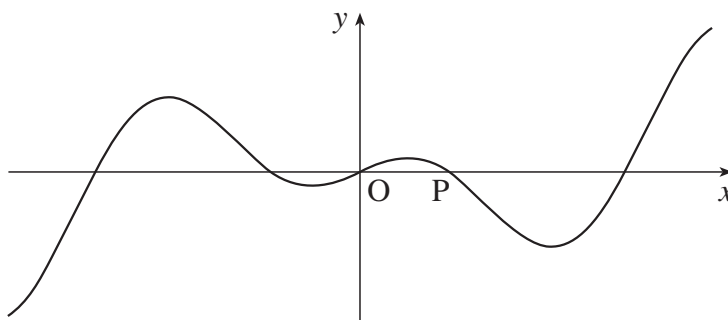


Fig. 8

- (i) Find the exact coordinates of P. [3]
- (ii) Show algebraically that $x \cos 2x$ is an odd function, and interpret this result graphically. [3]
- (iii) Find $\frac{dy}{dx}$. [2]
- (iv) Show that turning points occur on the curve for values of x which satisfy the equation $x \tan 2x = \frac{1}{2}$. [2]
- (v) Find the gradient of the curve at the origin.
- Show that the second derivative of $x \cos 2x$ is zero when $x = 0$. [4]
- (vi) Evaluate $\int_0^{\frac{1}{4}\pi} x \cos 2x dx$, giving your answer in terms of π . Interpret this result graphically. [6]