

ADVANCED GCE
MATHEMATICS
Core Mathematics 3

4723

Candidates answer on the Answer Booklet

OCR Supplied Materials:

- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:

- Scientific or graphical calculator

Wednesday 9 June 2010
Afternoon

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1 Find $\frac{dy}{dx}$ in each of the following cases:

(i) $y = x^3 e^{2x}$, [2]

(ii) $y = \ln(3 + 2x^2)$, [2]

(iii) $y = \frac{x}{2x+1}$. [2]

2 The transformations R, S and T are defined as follows.

R : reflection in the x -axis

S : stretch in the x -direction with scale factor 3

T : translation in the positive x -direction by 4 units

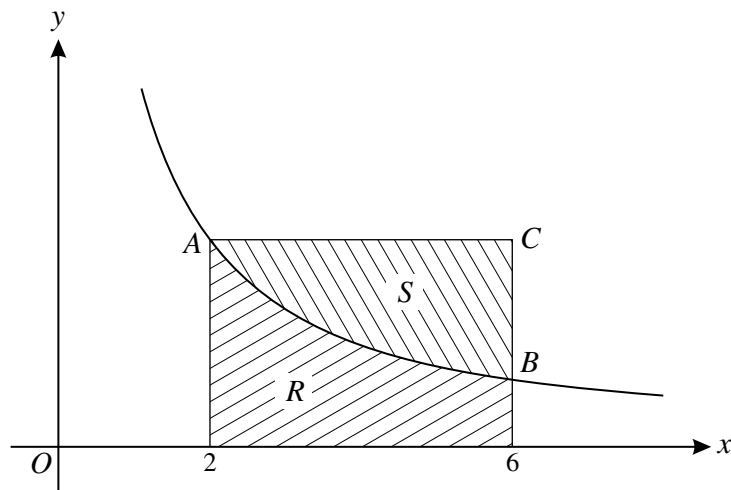
(i) The curve $y = \ln x$ is transformed by R followed by T. Find the equation of the resulting curve. [2]

(ii) Find, in terms of S and T, a sequence of transformations that transforms the curve $y = x^3$ to the curve $y = (\frac{1}{9}x - 4)^3$. You should make clear the order of the transformations. [2]

3 (i) Express the equation $\operatorname{cosec} \theta(3 \cos 2\theta + 7) + 11 = 0$ in the form $a \sin^2 \theta + b \sin \theta + c = 0$, where a , b and c are constants. [3]

(ii) Hence solve, for $-180^\circ < \theta < 180^\circ$, the equation $\operatorname{cosec} \theta(3 \cos 2\theta + 7) + 11 = 0$. [3]

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The diagram shows part of the curve $y = \frac{k}{x}$, where k is a positive constant. The points A and B on the curve have x -coordinates 2 and 6 respectively. Lines through A and B parallel to the axes as shown meet at the point C. The region R is bounded by the curve and the lines $x = 2$, $x = 6$ and $y = 0$. The region S is bounded by the curve and the lines AC and BC. It is given that the area of the region R is $\ln 81$.

(i) Show that $k = 4$. [3]

(ii) Find the exact volume of the solid produced when the region S is rotated completely about the x -axis. [4]

5 (i) Solve the inequality $|2x + 1| \leq |x - 3|$. [5]

(ii) Given that x satisfies the inequality $|2x + 1| \leq |x - 3|$, find the greatest possible value of $|x + 2|$. [2]

6 (i) Show by calculation that the equation

$$\tan^2 x - x - 2 = 0,$$

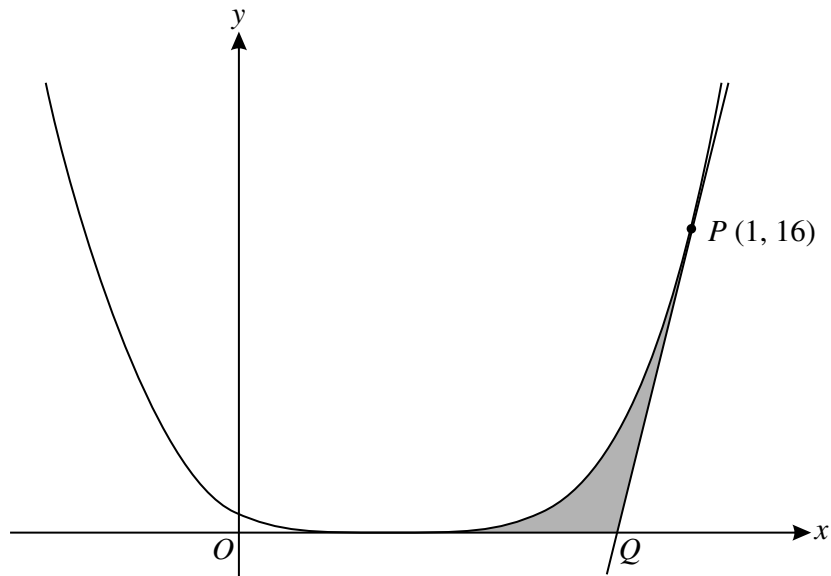
where x is measured in radians, has a root between 1.0 and 1.1. [3]

(ii) Use the iteration formula $x_{n+1} = \tan^{-1} \sqrt{2 + x_n}$ with a suitable starting value to find this root correct to 5 decimal places. You should show the outcome of each step of the process. [4]

(iii) Deduce a root of the equation

$$\sec^2 2x - 2x - 3 = 0. [3]$$

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The diagram shows the curve with equation $y = (3x - 1)^4$. The point P on the curve has coordinates $(1, 16)$ and the tangent to the curve at P meets the x -axis at the point Q . The shaded region is bounded by PQ , the x -axis and that part of the curve for which $\frac{1}{3} \leq x \leq 1$. Find the exact area of this shaded region. [10]

8 (i) Express $3 \cos x + 3 \sin x$ in the form $R \cos(x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. [3]

(ii) The expression $T(x)$ is defined by $T(x) = \frac{8}{3 \cos x + 3 \sin x}$.

(a) Determine a value of x for which $T(x)$ is not defined. [2]

(b) Find the smallest positive value of x satisfying $T(3x) = \frac{8}{9}\sqrt{6}$, giving your answer in an exact form. [4]

[Question 9 is printed overleaf.]

9 The functions f and g are defined for all real values of x by

$$f(x) = 4x^2 - 12x \quad \text{and} \quad g(x) = ax + b,$$

where a and b are non-zero constants.

- (i) Find the range of f . [3]
- (ii) Explain why the function f has no inverse. [2]
- (iii) Given that $g^{-1}(x) = g(x)$ for all values of x , show that $a = -1$. [4]
- (iv) Given further that $gf(x) < 5$ for all values of x , find the set of possible values of b . [4]



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