

Thursday 21 June 2012 – Afternoon

**A2 GCE MATHEMATICS (MEI)**

**4753/01** Methods for Advanced Mathematics (C3)

**QUESTION PAPER**

Candidates answer on the Printed Answer Book.

**OCR supplied materials:**

- Printed Answer Book 4753/01
- MEI Examination Formulae and Tables (MF2)

**Other materials required:**

- Scientific or graphical calculator

**Duration:** 1 hour 30 minutes



**INSTRUCTIONS TO CANDIDATES**

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

**INFORMATION FOR CANDIDATES**

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

**INSTRUCTION TO EXAMS OFFICER/INVIGILATOR**

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

## Section A (36 marks)

1 Show that  $\int_1^2 \frac{1}{\sqrt{3x-2}} dx = \frac{2}{3}$ . [5]

2 Solve the inequality  $|2x + 1| > 4$ . [3]

3 Find the gradient at the point  $(0, \ln 2)$  on the curve with equation  $e^{2y} = 5 - e^{-x}$ . [4]

4 Fig. 4 shows the curve  $y = f(x)$ , where  $f(x) = \sqrt{1 - 9x^2}$ ,  $-a \leq x \leq a$ .

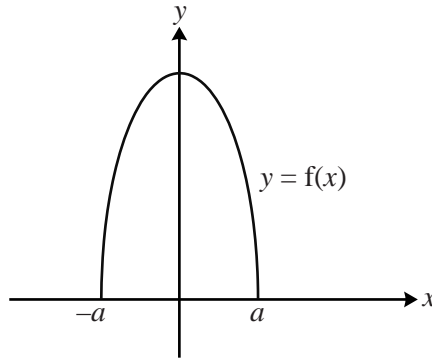


Fig. 4

(i) Find the value of  $a$ . [2]

(ii) Write down the range of  $f(x)$ . [1]

(iii) Sketch the curve  $y = f(\frac{1}{3}x) - 1$ . [3]

5 A termites' nest has a population of  $P$  million.  $P$  is modelled by the equation  $P = 7 - 2e^{-kt}$ , where  $t$  is in years, and  $k$  is a positive constant.

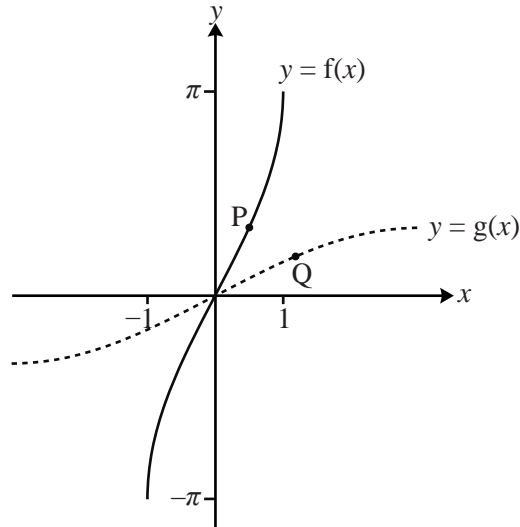
(i) Calculate the population when  $t = 0$ , and the long-term population, given by this model. [3]

(ii) Given that the population when  $t = 1$  is estimated to be 5.5 million, calculate the value of  $k$ . [3]

- 6 Fig. 6 shows the curve  $y = f(x)$ , where  $f(x) = 2\arcsin x$ ,  $-1 \leq x \leq 1$ .

Fig. 6 also shows the curve  $y = g(x)$ , where  $g(x)$  is the inverse function of  $f(x)$ .

P is the point on the curve  $y = f(x)$  with  $x$ -coordinate  $\frac{1}{2}$ .



**Fig. 6**

- (i) Find the  $y$ -coordinate of P, giving your answer in terms of  $\pi$ . [2]

The point Q is the reflection of P in  $y = x$ .

- (ii) Find  $g(x)$  and its derivative  $g'(x)$ . Hence determine the exact gradient of the curve  $y = g(x)$  at the point Q.

Write down the exact gradient of  $y = f(x)$  at the point P. [6]

- 7 You are given that  $f(x)$  and  $g(x)$  are odd functions, defined for  $x \in \mathbb{R}$ .

- (i) Given that  $s(x) = f(x) + g(x)$ , prove that  $s(x)$  is an odd function. [2]

- (ii) Given that  $p(x) = f(x)g(x)$ , determine whether  $p(x)$  is odd, even or neither. [2]

## Section B (36 marks)

- 8 Fig. 8 shows a sketch of part of the curve  $y = x \sin 2x$ , where  $x$  is in radians.

The curve crosses the  $x$ -axis at the point P. The tangent to the curve at P crosses the  $y$ -axis at Q.

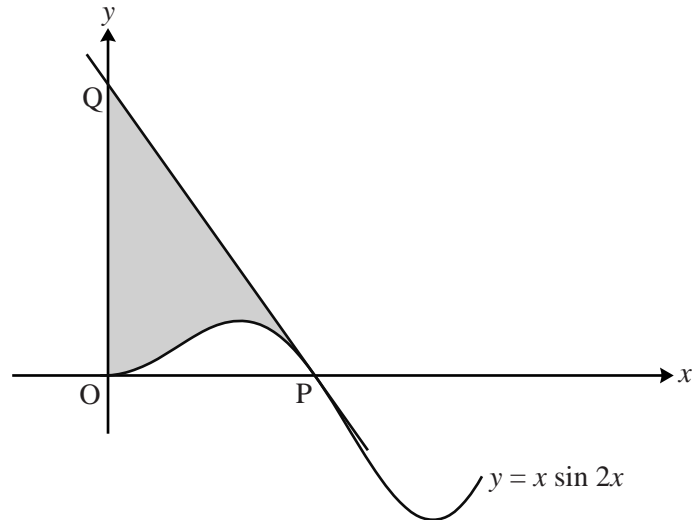


Fig. 8

- (i) Find  $\frac{dy}{dx}$ . Hence show that the  $x$ -coordinates of the turning points of the curve satisfy the equation  $\tan 2x + 2x = 0$ . [4]

- (ii) Find, in terms of  $\pi$ , the  $x$ -coordinate of the point P.

Show that the tangent PQ has equation  $2\pi x + 2y = \pi^2$ .

Find the exact coordinates of Q. [7]

- (iii) Show that the exact value of the area shaded in Fig. 8 is  $\frac{1}{8}\pi(\pi^2 - 2)$ . [7]

- 9 Fig. 9 shows the curve  $y = f(x)$ , which has a  $y$ -intercept at  $P(0, 3)$ , a minimum point at  $Q(1, 2)$ , and an asymptote  $x = -1$ .

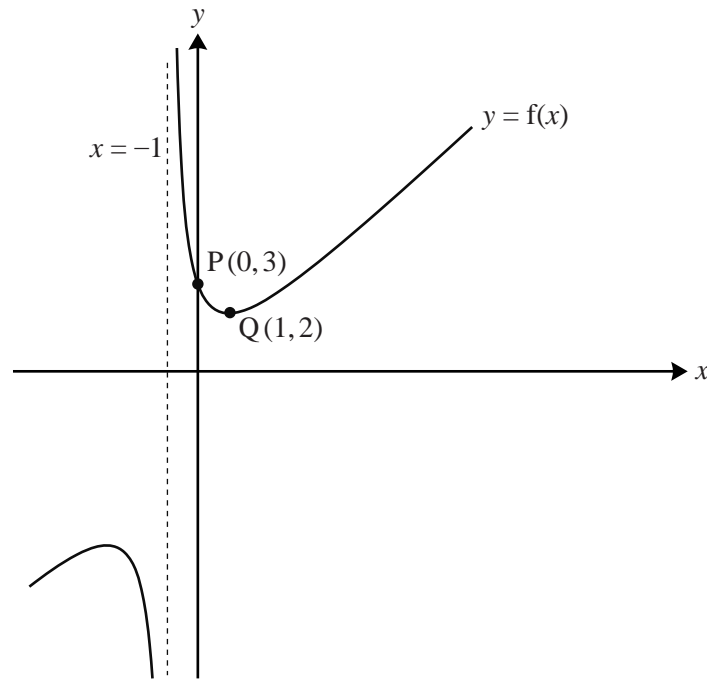


Fig. 9

- (i) Find the coordinates of the images of the points P and Q when the curve  $y = f(x)$  is transformed to

(A)  $y = 2f(x)$ ,

(B)  $y = f(x + 1) + 2$ .

[4]

You are now given that  $f(x) = \frac{x^2 + 3}{x + 1}$ ,  $x \neq -1$ .

- (ii) Find  $f'(x)$ , and hence find the coordinates of the other turning point on the curve  $y = f(x)$ .

[6]

- (iii) Show that  $f(x - 1) = x - 2 + \frac{4}{x}$ .

[3]

- (iv) Find  $\int_a^b \left(x - 2 + \frac{4}{x}\right) dx$  in terms of  $a$  and  $b$ .

Hence, by choosing suitable values for  $a$  and  $b$ , find the exact area enclosed by the curve  $y = f(x)$ , the  $x$ -axis, the  $y$ -axis and the line  $x = 1$ .

[5]

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