Maxima and minima

Question Paper 1

Level	A Level
Subject	Mathematics (Pure)
Exam Board	AQA
Module	Core 2
Topic	Calculus
Sub Topic	Maxima and minima
Booklet	Question Paper 1

Time Allowed: **79 minutes**

Score: **/66**

Percentage: **/100**

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

1 The point $P(2, 8)$ lies on a curve, and the point M is the only stationary point of the curve.

The curve has equation $y = 6 + 2x - \dfrac{8}{x^2}$.

(a) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$.

(3)

(b) Show that the normal to the curve at the point $P(2, 8)$ has equation $x + 4y = 34$.

(3)

(c) (i) Show that the stationary point M lies on the x-axis.

(3)

(ii) Hence **write down** the equation of the tangent to the curve at M.

(1)

(d) The tangent to the curve at M and the normal to the curve at P intersect at the point T. Find the coordinates of T.

(2)

(Total 12 marks)

2 A curve has the equation

$$y = \frac{12 + x^2\sqrt{x}}{x}, \quad x > 0$$

(a) Express $\dfrac{12 + x^2\sqrt{x}}{x}$ in the form $12x^p + x^q$.

(3)

(b) (i) Hence find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$.

(2)

(ii) Find an equation of the normal to the curve at the point on the curve where $x = 4$.

(4)

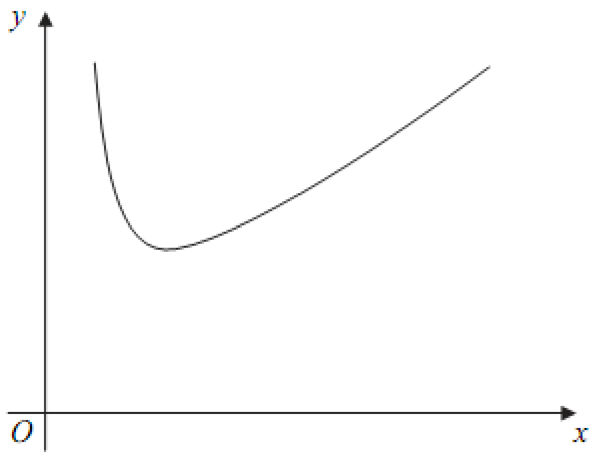
(iii) The curve has a stationary point P. Show that the x-coordinate of P can be written in the form 2^k, where k is a rational number.

(3)

(Total 12 marks)

3

A curve C is defined for $x > 0$ by the equation $y = x + 3 + \dfrac{8}{x^4}$ and is sketched below.



(a) Given that $y = x + 3 + \dfrac{8}{x^4}$, find $\dfrac{dy}{dx}$.

(3)

(b) Find an equation of the tangent at the point on the curve C where $x = 1$.

(3)

(c) The curve C has a minimum point M. Find the coordinates of M.

(4)

(d) (i) Find $\displaystyle\int \left(x + 3 + \frac{8}{x^4} \right) dx$.

(3)

(ii) Hence find the area of the region bounded by the curve C, the x-axis and the lines $x = 1$ and $x = 2$.

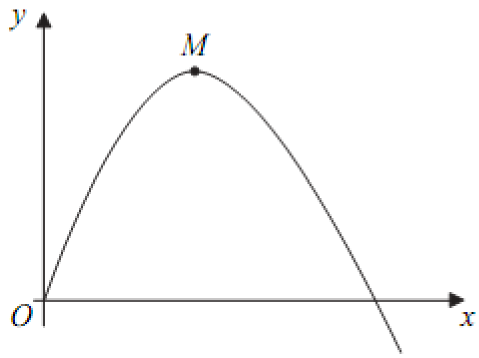
(2)

(e) The curve C is translated by $\begin{bmatrix} 0 \\ k \end{bmatrix}$ to give the curve $y = \text{f}(x)$. Given that the x-axis is a tangent to the curve $y = \text{f}(x)$, state the value of the constant k.

(1)

(Total 16 marks)

4 The diagram shows part of a curve with a maximum point M.



The curve is defined for $x \geq 0$ by the equation

$$y = 6x - 2x^{\frac{3}{2}}$$

(a) Find $\frac{dy}{dx}$.

(3)

(b) (i) Hence find the coordinates of the maximum point M.

(3)

(ii) Write down the equation of the normal to the curve at M.

(1)

(c) The point $P\left(\frac{9}{4}, \frac{27}{4}\right)$ lies on the curve.

(i) Find an equation of the normal to the curve at the point P, giving your answer in the form $ax + by = c$, where a, b and c are positive integers.

(4)

(ii) The normals to the curve at the points M and P intersect at the point R. Find the coordinates of R.

(2)
(Total 13 marks)

5 A curve C has the equation

$$y = \frac{x^3 + \sqrt{x}}{x}, x > 0$$

(a) Express $\frac{x^3 + \sqrt{x}}{x}$ in the form $x^p + x^q$.

(3)

(b) (i) Hence find $\frac{dy}{dx}$.

(2)

(ii) Find an equation of the normal to the curve C at the point on the curve where $x = 1$.

(4)

(c) (i) Find $\frac{d^2y}{dx^2}$.

(2)

(ii) Hence deduce that the curve C has no maximum points.

(2)
(Total 13 marks)