Maxima and Minima

Question Paper 1

Level	A Level
Subject	Mathematics (Pure)
Exam Board	AQA
Module	Core 1
Topic	Calculus
Sub Topic	Maxima and minima
Booklet	Question Paper 1

Time Allowed: **84 minutes**

Score: **/70**

Percentage: **/100**

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

1 A bird flies from a tree. At time t seconds, the bird's height, y metres, above the horizontal ground is given by

$$y = \frac{1}{8}t^4 - t^2 + 5, \quad 0 \le t \le 4$$

(a) Find $\frac{dy}{dt}$.

(2)

(b) (i) Find the rate of change of height of the bird in metres per second when $t = 1$.

(2)

(ii) Determine, with a reason, whether the bird's height above the horizontal ground is increasing or decreasing when $t = 1$.

(1)

(c) (i) Find the value of $\frac{d^2y}{dt^2}$ when $t = 2$.

(2)

(ii) Given that y has a stationary value when $t = 2$, state whether this is a maximum value or a minimum value.

(1)

(Total 8 marks)

2 (a) The polynomial f(x) is given by f(x)= $x^3 - 4x + 15$.

(i) Use the Factor Theorem to show that $x + 3$ is a factor of f(x).

(2)

(ii) Express f(x) in the form $(x + 3)(x^2 + px + q)$, where p and q are integers.

(2)

(b) A curve has equation $y = x^4 - 8x^2 + 60x + 7$.

(i) Find $\frac{dy}{dx}$.

(3)

(ii) Show that the x-coordinates of any stationary points of the curve satisfy the equation

$$x^3 - 4x + 15 = 0$$

(1)

(iii) Use the results above to show that the only stationary point of the curve occurs when $x = -3$.

(2)

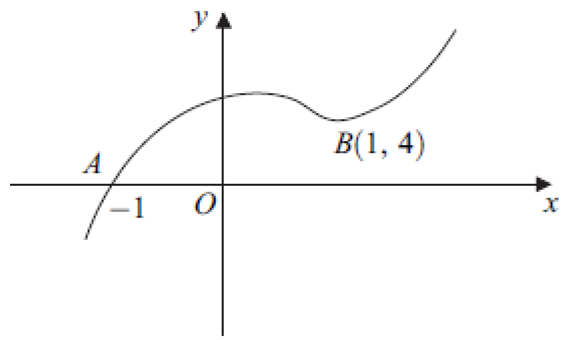
(iv) Find the value of $\frac{d^2y}{dx^2}$ when $x = -3$.

(3)

(v) Hence determine, with a reason, whether the curve has a maximum point or a minimum point when $x = -3$.

(1)
(Total 14 marks)

3 The curve with equation $y = x^5 - 3x^2 + x + 5$ is sketched below. The point O is at the origin and the curve passes through the points $A(-1, 0)$ and $B(1, 4)$.



(a) Given that $y = x^5 - 3x^2 + x + 5$, find:

(i) $\frac{dy}{dx}$;

(3)

(ii) $\frac{d^2y}{dx^2}$.

(1)

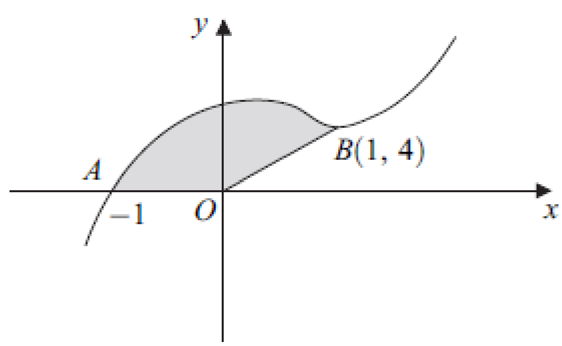
(b) Find an equation of the tangent to the curve at the point $A(-1, 0)$.

(2)

(c) Verify that the point B, where $x = 1$, is a minimum point of the curve.

(3)

(d) The curve with equation $y = x^5 - 3x^2 + x + 5$ is sketched below. The point O is at the origin and the curve passes through the points $A(-1, 0)$ and $B(1, 4)$.



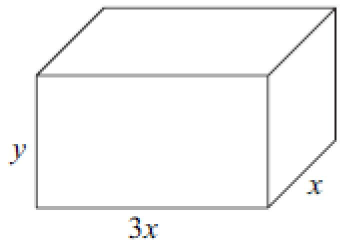
(i) Find $\int_{-1}^{1} (x^5 - 3x^2 + x + 5)\,dx$.

(5)

(ii) Hence find the area of the shaded region bounded by the curve between A and B and the line segments AO and OB.

(2)
(Total 16 marks)

4 The diagram shows a solid cuboid with sides of lengths x cm, $3x$ cm and y cm.



The total surface area of the cuboid is 32 cm^2.

(a) (i) Show that $3x^2 + 4xy = 16$.

(2)

(ii) Hence show that the volume, V cm^3, of the cuboid is given by

$$V = 12x - \frac{9x^3}{4}$$

(2)

(b) Find $\frac{dV}{dx}$.

(2)

(c) (i) Verify that a stationary value of V occurs when $x = \frac{4}{3}$.

(2)

(ii) Find $\frac{d^2V}{dx^2}$ and hence determine whether V has a maximum value or a minimum value when $x = \frac{4}{3}$.

(2)
(Total 10 marks)

5 The curve with equation $y = 13 + 18x + 3x^2 - 4x^3$ passes through the point P where $x = -1$.

(a) Find $\frac{dy}{dx}$

(3)

(b) Show that the point P is a stationary point of the curve and find the other value of x where the curve has a stationary point

(3)

(c) (i) Find the value of $\frac{d^2y}{dx^2}$ at the point P.

(3)

(ii) Hence, or otherwise, determine whether P is a maximum point or a minimum point.

(1)
(Total 10 marks)

6 The depth of water, y metres, in a tank after time t hours is given by

$$y = \frac{1}{8}t^4 - 2t^2 + 4t, \qquad 0 \le t \le 4$$

(a) Find:

(i) $\frac{dy}{dt}$;

(3)

(ii) $\frac{d^2y}{dt^2}$.

(2)

(b) Verify that y has a stationary value when $t = 2$ and determine whether it is a maximum value or a minimum value.

(4)

(c) (i) Find the rate of change of the depth of water, in metres per hour, when $t = 1$.

(2)

(ii) Hence determine, with a reason, whether the depth of water is increasing or decreasing when $t = 1$.

(1)
(Total 12 marks)