

# Straight Lines

## Question Paper 4

Level	A Level
Subject	Mathematics (Pure)
Exam Board	AQA
Module	Core 1
Topic	Co-ordinate geometry
Sub Topic	Straight lines
Booklet	Question Paper 4

**Time Allowed:** 81 minutes

**Score:** /68

**Percentage:** /100

**Grade Boundaries:**

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

**1**

A circle with centre  $C$  has equation  $x^2 + y^2 - 6x + 10y + 9 = 0$ .

- (a) Express this equation in the form

$$(x - a)^2 + (y - b)^2 = r^2 \quad (3)$$

- (b) Write down:

(i) the coordinates of  $C$ ;

(ii) the radius of the circle.

(2)

- (c) The point  $D$  has coordinates  $(7, -2)$ .

(i) Verify that the point  $D$  lies on the circle.

(1)

(ii) Find an equation of the normal to the circle at the point  $D$ , giving your answer in the form  $mx + ny = p$ , where  $m$ ,  $n$  and  $p$  are integers.

(3)

- (d) (i) A line has equation  $y = kx$ . Show that the  $x$ -coordinates of any points of intersection of the line and the circle satisfy the equation

$$(k^2 + 1)x^2 + 2(5k - 3)x + 9 = 0 \quad (2)$$

(ii) Find the values of  $k$  for which the equation

$$(k^2 + 1)x^2 + 2(5k - 3)x + 9 = 0$$

has equal roots.

(5)

(iii) Describe the geometrical relationship between the line and the circle when  $k$  takes either of the values found in part (d)(ii).

(1)

**(Total 17 marks)**

**2**

The line  $AB$  has equation  $3x + 5y = 11$ .

- (a) (i) Find the gradient of  $AB$ .

(2)

- (ii) The point  $A$  has coordinates  $(2, 1)$ . Find an equation of the line which passes through the point  $A$  and which is perpendicular to  $AB$ .

(3)

- (b) The line  $AB$  intersects the line with equation  $2x + 3y = 8$  at the point  $C$ . Find the coordinates of  $C$ .

(3)

(Total 8 marks)

3

A circle with centre  $C$  has equation

$$(x - 5)^2 + (y + 12)^2 = 169$$

- (a) Write down:

- (i) the coordinates of  $C$ ;

(1)

- (ii) the radius of the circle.

(1)

- (b) (i) Verify that the circle passes through the origin  $O$ .

(1)

- (ii) Given that the circle also passes through the points  $(10, 0)$  and  $(0, p)$ , sketch the circle and find the value of  $p$ .

(3)

- (c) The point  $A(-7, -7)$  lies on the circle.

- (i) Find the gradient of  $AC$ .

(2)

- (ii) Hence find an equation of the tangent to the circle at the point  $A$ , giving your answer in the form  $ax + by + c = 0$ , where  $a$ ,  $b$  and  $c$  are integers.

(3)

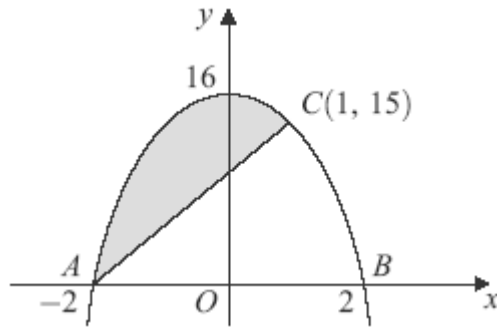
(Total 11 marks)

- 4** (a) (i) Express  $x^2 - 8x + 17$  in the form  $(x - p)^2 + q$ , where  $p$  and  $q$  are integers. (2)
- (ii) Hence write down the minimum value of  $x^2 - 8x + 17$ . (1)
- (iii) State the value of  $x$  for which the minimum value of  $x^2 - 8x + 17$  occurs. (1)
- (b) The point  $A$  has coordinates  $(5, 4)$  and the point  $B$  has coordinates  $(x, 7 - x)$ .
- (i) Expand  $(x - 5)^2$ . (1)
- (ii) Show that  $AB^2 = 2(x^2 - 8x + 17)$ . (3)
- (iii) Use your results from part (a) to find the minimum value of the distance  $AB$  as  $x$  varies. (2)
- (Total 10 marks)**

- 5** The triangle  $ABC$  has vertices  $A(-2, 3)$ ,  $B(4, 1)$  and  $C(2, -5)$ .
- (a) Find the coordinates of the mid-point of  $BC$ . (2)
- (b) (i) Find the gradient of  $AB$ , in its simplest form. (2)
- (ii) Hence find an equation of the line  $AB$ , giving your answer in the form  $x + qy = r$ , where  $q$  and  $r$  are integers. (2)
- (iii) Find an equation of the line passing through  $C$  which is parallel to  $AB$ . (2)
- (c) Prove that angle  $ABC$  is a right angle. (3)
- (Total 11 marks)**

**6**

The curve with equation  $y = 16 - x^4$  is sketched below.



The points  $A(-2, 0)$ ,  $B(2, 0)$  and  $C(1, 15)$  lie on the curve.

(a) Find an equation of the straight line  $AC$ .

**(3)**

(b) (i) Find  $\int_{-2}^1 (16 - x^4) dx$ .

**(5)**

(ii) Hence calculate the area of the shaded region bounded by the curve and the line  $AC$ .

**(3)**

**(Total 11 marks)**