

Straight Lines

Question Paper 4

Level	A Level
Subject	Mathematics (Pure)
Exam Board	AQA
Module	Core 1
Topic	Co-ordinate geometry
Sub Topic	Straight lines
Booklet	Question Paper 4

Time Allowed: 81 minutes

Score: /68

Percentage: /100

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

1

A circle with centre C has equation $x^2 + y^2 - 6x + 10y + 9 = 0$.

- (a) Express this equation in the form

$$(x - a)^2 + (y - b)^2 = r^2 \quad (3)$$

- (b) Write down:

(i) the coordinates of C ;

(ii) the radius of the circle.

(2)

- (c) The point D has coordinates $(7, -2)$.

(i) Verify that the point D lies on the circle.

(1)

(ii) Find an equation of the normal to the circle at the point D , giving your answer in the form $mx + ny = p$, where m , n and p are integers.

(3)

- (d) (i) A line has equation $y = kx$. Show that the x -coordinates of any points of intersection of the line and the circle satisfy the equation

$$(k^2 + 1)x^2 + 2(5k - 3)x + 9 = 0 \quad (2)$$

(ii) Find the values of k for which the equation

$$(k^2 + 1)x^2 + 2(5k - 3)x + 9 = 0$$

has equal roots.

(5)

(iii) Describe the geometrical relationship between the line and the circle when k takes either of the values found in part (d)(ii).

(1)

(Total 17 marks)

2

The line AB has equation $3x + 5y = 11$.

- (a) (i) Find the gradient of AB .

(2)

- (ii) The point A has coordinates $(2, 1)$. Find an equation of the line which passes through the point A and which is perpendicular to AB .

(3)

- (b) The line AB intersects the line with equation $2x + 3y = 8$ at the point C . Find the coordinates of C .

(3)

(Total 8 marks)

3

A circle with centre C has equation

$$(x - 5)^2 + (y + 12)^2 = 169$$

- (a) Write down:

- (i) the coordinates of C ;

(1)

- (ii) the radius of the circle.

(1)

- (b) (i) Verify that the circle passes through the origin O .

(1)

- (ii) Given that the circle also passes through the points $(10, 0)$ and $(0, p)$, sketch the circle and find the value of p .

(3)

- (c) The point $A(-7, -7)$ lies on the circle.

- (i) Find the gradient of AC .

(2)

- (ii) Hence find an equation of the tangent to the circle at the point A , giving your answer in the form $ax + by + c = 0$, where a , b and c are integers.

(3)

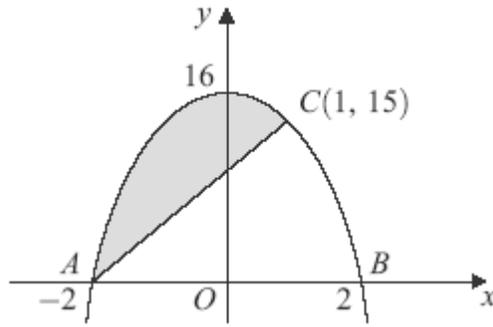
(Total 11 marks)

- 4** (a) (i) Express $x^2 - 8x + 17$ in the form $(x - p)^2 + q$, where p and q are integers. (2)
- (ii) Hence write down the minimum value of $x^2 - 8x + 17$. (1)
- (iii) State the value of x for which the minimum value of $x^2 - 8x + 17$ occurs. (1)
- (b) The point A has coordinates $(5, 4)$ and the point B has coordinates $(x, 7 - x)$.
- (i) Expand $(x - 5)^2$. (1)
- (ii) Show that $AB^2 = 2(x^2 - 8x + 17)$. (3)
- (iii) Use your results from part (a) to find the minimum value of the distance AB as x varies. (2)
- (Total 10 marks)**

- 5** The triangle ABC has vertices $A(-2, 3)$, $B(4, 1)$ and $C(2, -5)$.
- (a) Find the coordinates of the mid-point of BC . (2)
- (b) (i) Find the gradient of AB , in its simplest form. (2)
- (ii) Hence find an equation of the line AB , giving your answer in the form $x + qy = r$, where q and r are integers. (2)
- (iii) Find an equation of the line passing through C which is parallel to AB . (2)
- (c) Prove that angle ABC is a right angle. (3)
- (Total 11 marks)**

6

The curve with equation $y = 16 - x^4$ is sketched below.



The points $A(-2, 0)$, $B(2, 0)$ and $C(1, 15)$ lie on the curve.

(a) Find an equation of the straight line AC .

(3)

(b) (i) Find $\int_{-2}^1 (16 - x^4) dx$.

(5)

(ii) Hence calculate the area of the shaded region bounded by the curve and the line AC .

(3)

(Total 11 marks)