Transformation of graphs

Question Paper 1

Level	A Level
Subject	Mathematics (Pure)
Exam Board	AQA
Module	Core 2
Topic	Algebra
Sub Topic	Transformation of graphs
Booklet	Question Paper 1

Time Allowed: **85 minutes**

Score: **/71**

Percentage: **/100**

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

1 (a) Write down the two solutions of the equation $\tan(x + 30°) = \tan 79°$ in the interval $0° \leq x \leq 360°$.

(2)

(b) Describe a single geometrical transformation that maps the graph of $y = \tan x$ onto the graph of $y = \tan(x + 30°)$.

(2)

(c) (i) Given that $5 + \sin^2 \theta = (5 + 3 \cos \theta) \cos \theta$, show that $\cos \theta = \frac{3}{4}$.

(5)

(ii) **Hence** solve the equation $5 + \sin^2 2x = (5 + 3 \cos 2x) \cos 2x$ in the interval $0 < x < 2\pi$, giving your values of x in radians to three significant figures.

(3)
(Total 12 marks)

2 (a) Use the trapezium rule with five ordinates (four strips) to find an approximate value for

$\int_0^2 \sqrt{8x^3 + 1}\, dx$, giving your answer to three significant figures.

(4)

(b) Describe the single transformation that maps the graph of $y = \sqrt{8x^3 + 1}$ onto the graph of $y = \sqrt{x^3 + 1}$.

(2)

(c) The curve with equation $y = \sqrt{x^3 + 1}$ is translated by $\begin{bmatrix} 2 \\ -0.7 \end{bmatrix}$ to give the curve with equation $y = g(x)$. Find the value of $g(4)$.

(3)
(Total 9 marks)

3 (a) (i) Describe the geometrical transformation that maps the graph of

$y = \left(1 + \frac{x}{3}\right)^6$ onto the graph of $y = (1 + 2x)^6$.

(2)

(ii) The curve $y = \left(1 + \frac{x}{3}\right)^6$ is translated by the vector $\begin{bmatrix} 3 \\ 0 \end{bmatrix}$ to give the curve $y = g(x)$.

Find an expression for $g(x)$, simplifying your answer.

(2)

(b) The first four terms in the binomial expansion of $\left(1 + \frac{x}{3}\right)^6$ are $1 + ax + bx^2 + cx^3$.

Find the values of the constants a, b and c, giving your answers in their simplest form.

(4)
(Total 8 marks)

4 (a) Use the trapezium rule with four ordinates (three strips) to find an approximate value for

$\int_0^{1.5} \sqrt{27x^3 + 4}\,dx$, giving your answer to three significant figures.

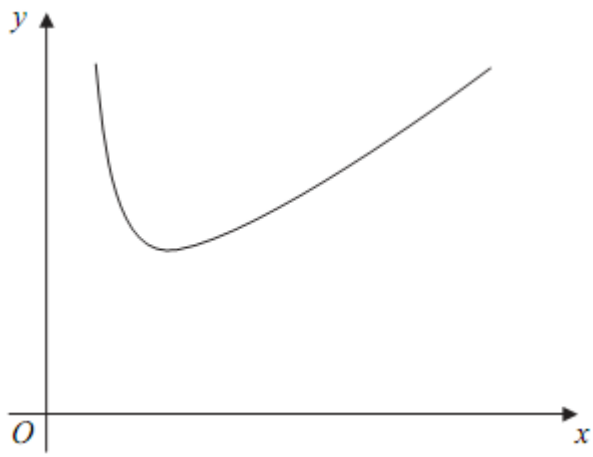
(4)

(b) The curve with equation $y = \sqrt{27x^3 + 4}$ is stretched parallel to the x-axis with scale factor 3 to give the curve with equation $y = g(x)$. Write down an expression for $g(x)$.

(2)

(Total 6 marks)

5 A curve C is defined for $x > 0$ by the equation $y = x + 3 + \frac{8}{x^4}$ and is sketched below.



(a) Given that $y = x + 3 + \frac{8}{x^4}$, find $\frac{dy}{dx}$.

(3)

(b) Find an equation of the tangent at the point on the curve C where $x = 1$.

(3)

(c) The curve C has a minimum point M. Find the coordinates of M.

(4)

(d) (i) Find $\int \left(x + 3 + \frac{8}{x^4} \right) dx$.

(3)

(ii) Hence find the area of the region bounded by the curve C, the x-axis and the lines $x = 1$ and $x = 2$.

(2)

(e) The curve C is translated by $\begin{bmatrix} 0 \\ k \end{bmatrix}$ to give the curve $y = \mathrm{f}(x)$. Given that the x-axis is a tangent to the curve $y = \mathrm{f}(x)$, state the value of the constant k.

(1)
(Total 16 marks)

6 (a) Sketch the curve with equation $y = 4^x$, indicating the coordinates of any point where the curve intersects the coordinate axes.

(2)

(b) Describe the geometrical transformation that maps the graph of $y = 4^x$ onto the graph of $y = 4^x - 5$.

(2)

(c) (i) Use the substitution $Y = 2^x$ to show that the equation $4^x - 2^{x+2} - 5 = 0$ can be written as $Y^2 - 4Y - 5 = 0$.

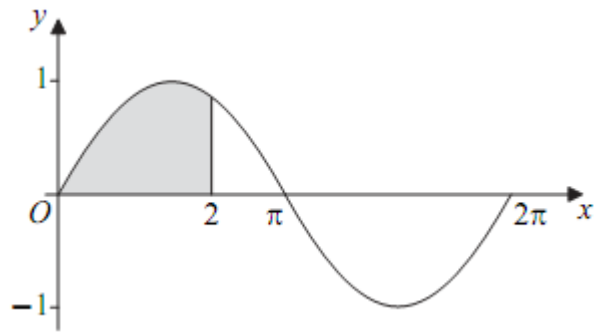
(2)

(ii) Hence show that the equation $4^x - 2^{x+2} - 5 = 0$ has only one real solution.

Use logarithms to find this solution, giving your answer to three decimal places.

(4)
(Total 10 marks)

7 A curve C, defined for $0 \le x \le 2\pi$ by the equation $y = \sin x$, where x is in radians, is sketched below. The region bounded by the curve C, the x-axis from 0 to 2 and the line $x = 2$ is shaded.



(a) The area of the shaded region is given by $\int_0^2 \sin x \, dx$, where x is in radians.

Use the trapezium rule with five ordinates (four strips) to find an approximate value for the area of the shaded region, giving your answer to three significant figures.

(4)

(b) Describe the geometrical transformation that maps the graph of $y = \sin x$ onto the graph of $y = 2 \sin x$.

(2)

(c) Use a trigonometrical identity to solve the equation

$$2 \sin x = \cos x$$

in the interval $0 \leq x \leq 2\pi$, giving your solutions in radians to three significant figures.

(4)
(Total 10 marks)