Transformation of graphs

Question Paper 1

Level	A Level
Subject	Mathematics (Pure)
Exam Board	AQA
Module	Core 3
Topic	Algebra
Sub Topic	Transformation of graphs
Booklet	Question Paper 1

Time Allowed: **89 minutes**

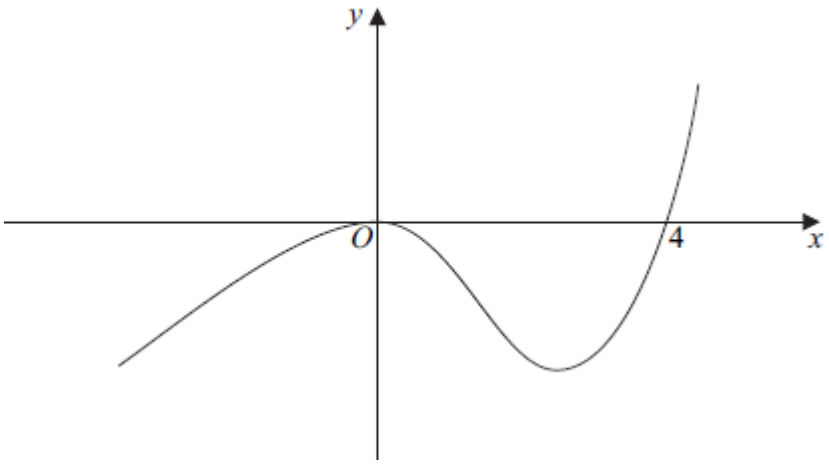
Score: **/75**

Percentage: **/100**

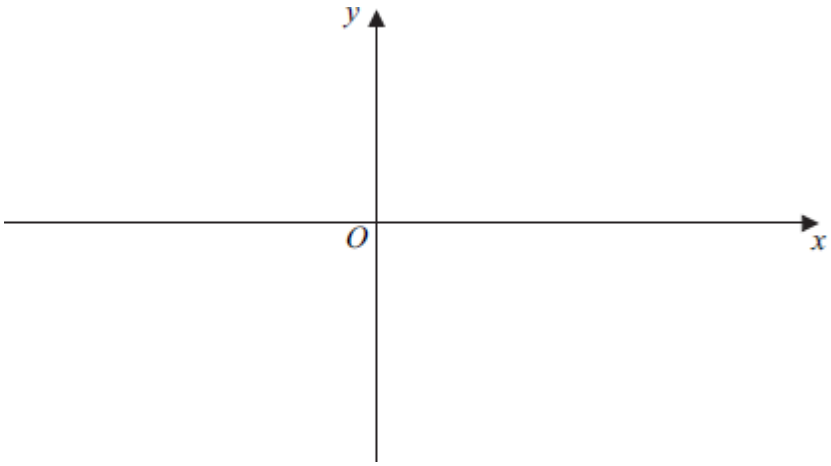
Grade Boundaries:

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

1 The diagram shows a sketch of the curve with equation $y = f(x)$.



(a) On the axes below, sketch the curve with equation $y = |f(x)|$.

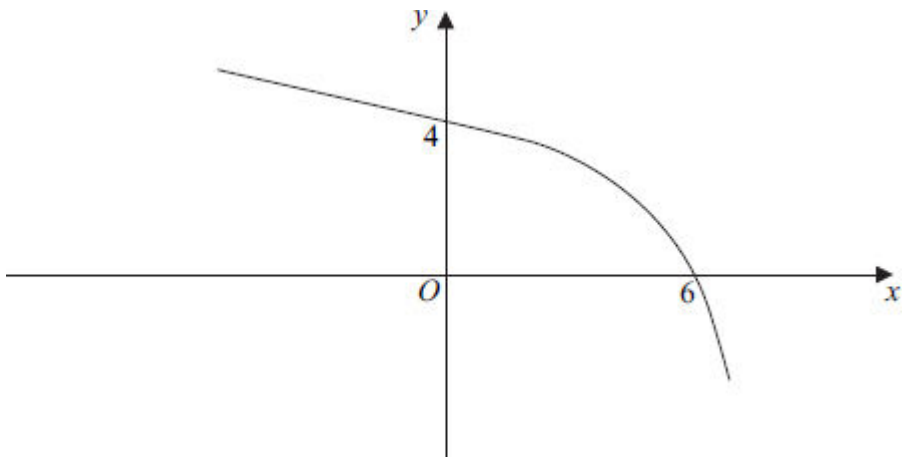


(2)

(b) Describe a sequence of two geometrical transformations that maps the graph of $y = f(x)$ onto the graph of $y = f(2x - 1)$.

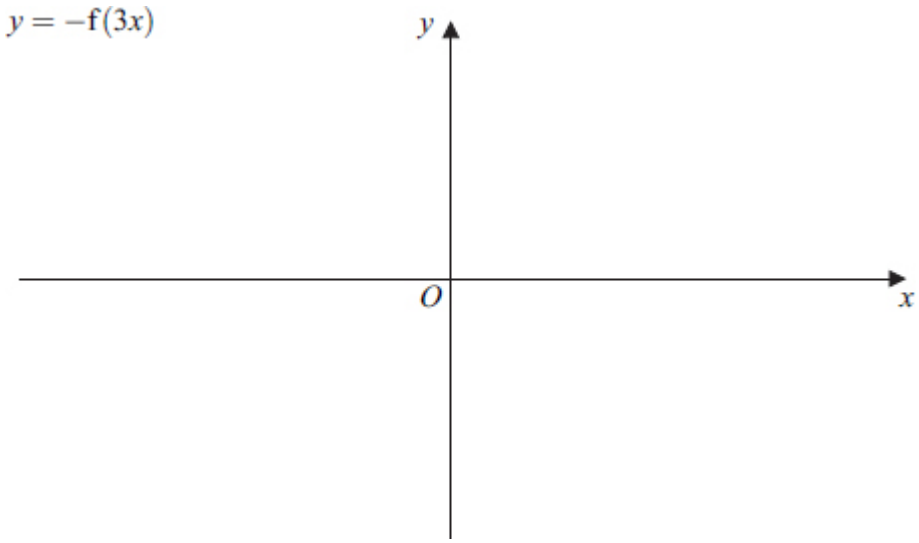
(4)
(Total 6 marks)

2 The diagram shows a sketch of the curve with equation $y = f(x)$.



(a) On **Figure 1**, below, sketch the curve with equation $y = -f(3x)$, indicating the values where the curve cuts the coordinate axes.

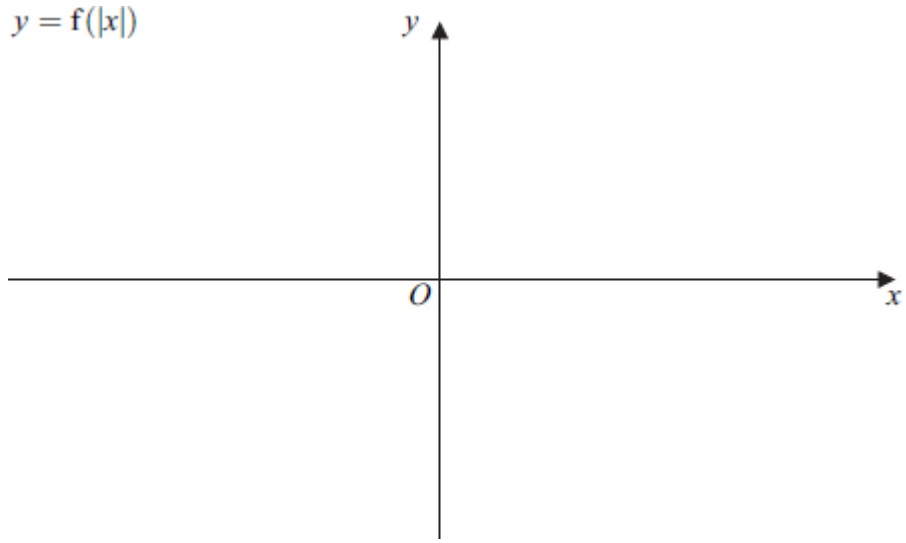
Figure 1



(2)

(b) On **Figure 2**, on the opposite page, sketch the curve with equation $y = \text{f}(|x|)$, indicating the values where the curve cuts the coordinate axes.

Figure 2

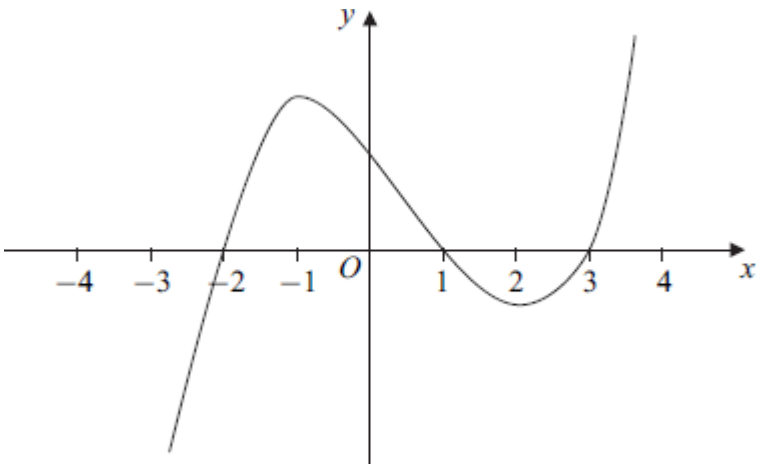


(3)

(c) Describe a sequence of two geometrical transformations that maps the graph of $y = \text{f}(x)$ onto the graph of $y = \text{f}\left(-\frac{1}{2}x\right)$.

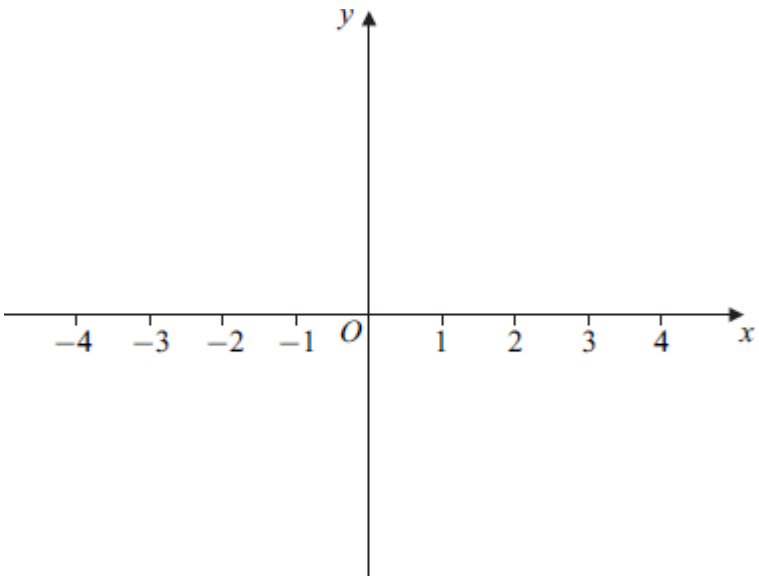
(4)
(Total 9 marks)

3 The sketch shows part of the curve with equation $y = f(x)$.



(a) On **Figure 1** below, sketch the curve with equation $y = |f(x)|$.

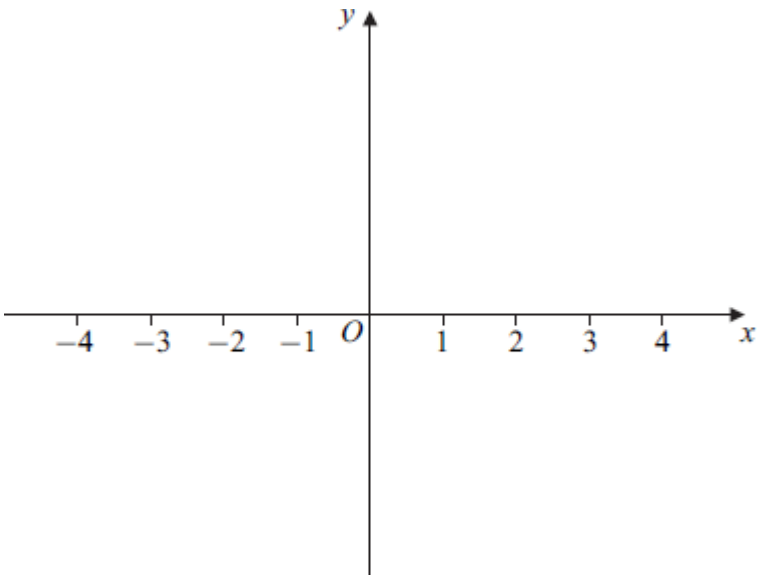
Figure 1



(3)

(b) On **Figure 2**, sketch the curve with equation $y = \text{f}(|x|)$.

Figure 2



(2)

(c) Describe a sequence of two geometrical transformations that maps the graph of $y = \text{f}(x)$ onto the graph of $y = \frac{1}{2}\text{f}(x + 1)$.

(4)

(d) The maximum point of the curve with equation $y = \text{f}(x)$ has coordinates $(-1, 10)$.
Find the coordinates of the maximum point of the curve with equation $y = \frac{1}{2}\text{f}(x + 1)$.

(2)
(Total 11 marks)

4 (a) (i) Solve the equation $\text{cosec}\theta = -4$ for $0° < \theta < 360°$, giving your answers to the nearest 0.1°.

(2)

(ii) Solve the equation

$$2\cot^2(2x + 30°) = 2 - 7\,\text{cosec}(2x + 30°)$$

for $0° < x < 180°$, giving your answers to the nearest 0.1°.

(6)

(b) Describe a sequence of two geometrical transformations that maps the graph of $y = \text{cosec}\,x$ onto the graph of $y = \text{cosec}(2x + 30°)$.

(4)
(Total 12 marks)

5

(a) Use the mid-ordinate rule with four strips to find an estimate for $\int_0^{12} \ln\left(x^2 + 5\right) dx$, giving your answer to three significant figures.

(4)

(b) A curve has equation $y = \ln(x^2 + 5)$.

(i) Show that this equation can be rewritten as $x^2 = e^y - 5$.

(1)

(ii) The region bounded by the curve, the lines $y = 5$ and $y = 10$ and the y-axis is rotated through 360° about the y-axis. Find the exact value of the volume of the solid generated.

(4)

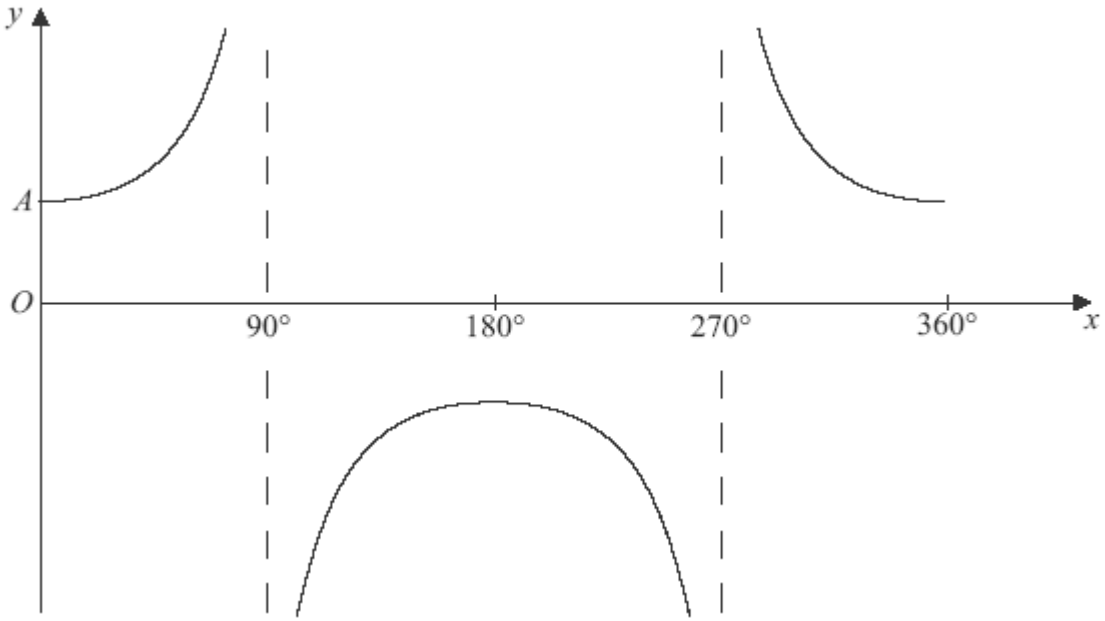
(c) The graph with equation $y = \ln(x^2 + 5)$ is stretched with scale factor 4 parallel to the x-axis, and then translated through $\begin{bmatrix} 0 \\ 3 \end{bmatrix}$ to give the graph with equation $y = f(x)$.

Write down an expression for $f(x)$.

(3)
(Total 12 marks)

6

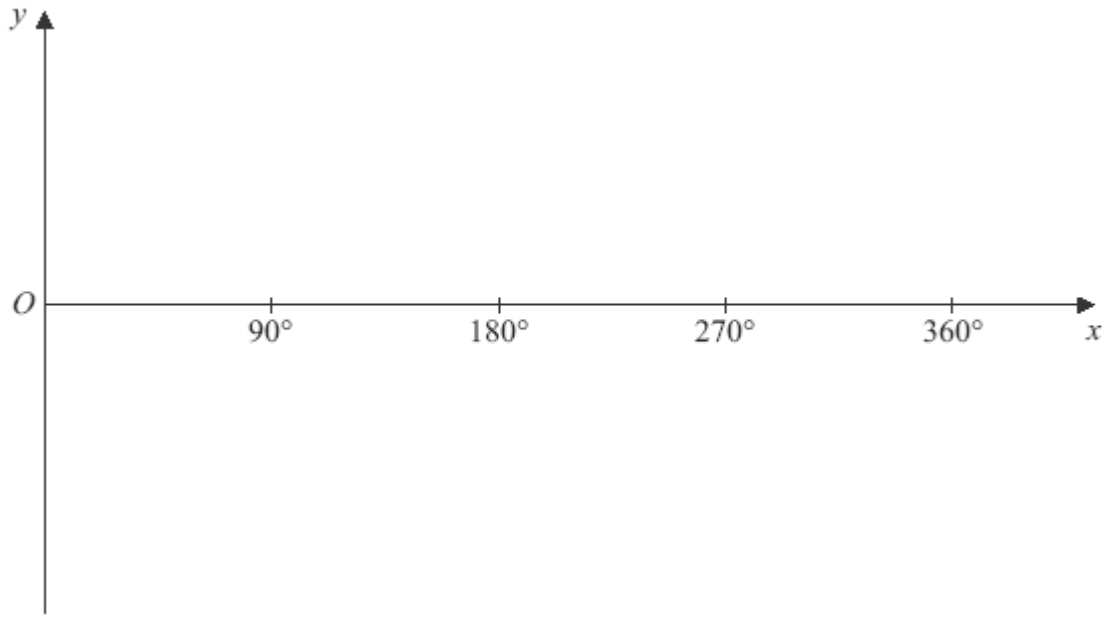
(a) The diagram shows the graph of $y = \sec x$ for $0° \leq x \leq 360°$.



(i) The point A on the curve is where $x = 0$. State the y-coordinate of A.

(1)

(ii) Sketch, on the axes below, the graph of $y = |\sec 2x|$ for $0° \leq x \leq 360°$.



(3)

(b) Solve the equation $\sec x = 2$, giving all values of x in degrees in the interval $0° \leq x \leq 360°$.

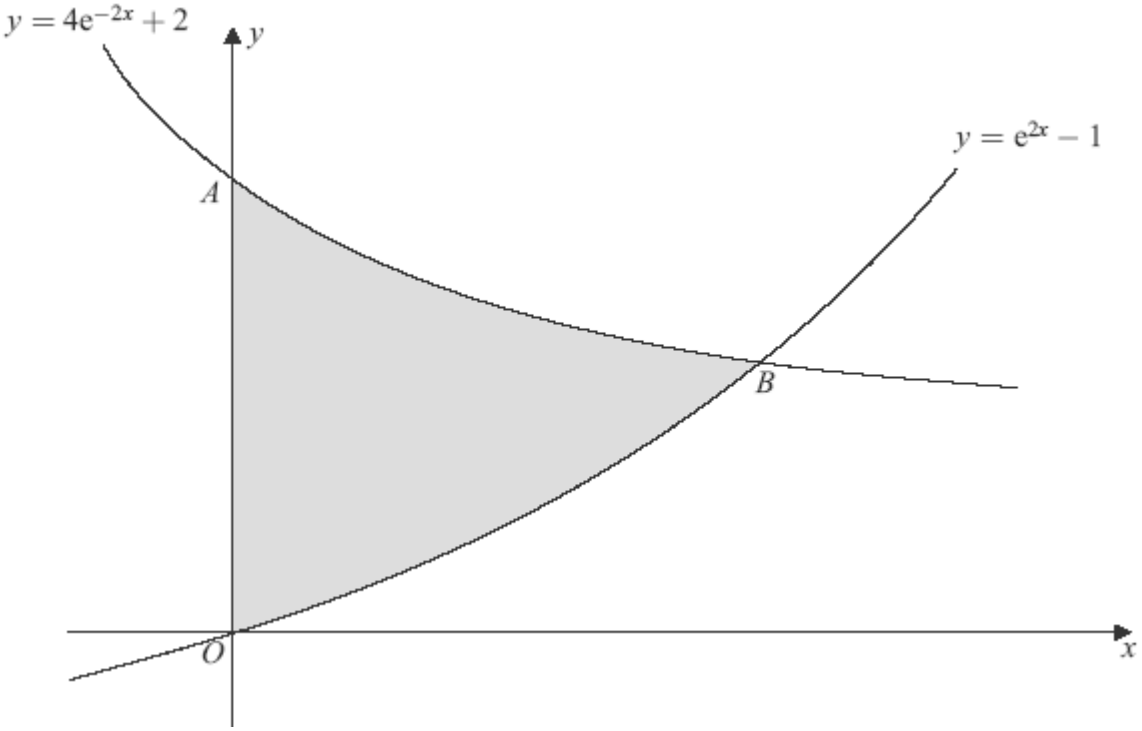
(2)

(c) Solve the equation $|\sec(2x - 10°)| = 2$, giving all values of x in degrees in the interval $0° \leq x \leq 180°$.

(4)

(Total 10 marks)

7 The diagram shows the curves $y = e^{2x} - 1$ and $y = 4e^{-2x} + 2$.



The curve $y = 4e^{-2x} + 2$ crosses the y-axis at the point A and the curves intersect at the point B.

(a) Describe a sequence of two geometrical transformations that maps the graph of $y = e^x$ onto the graph of $y = e^{2x} - 1$.

(4)

(b) Write down the coordinates of the point A.

(1)

(c) (i) Show that the x-coordinate of the point B satisfies the equation

$$(e^{2x})^2 - 3e^{2x} - 4 = 0$$

(2)

(ii) Hence find the exact value of the x-coordinate of the point B.

(3)

(d) Find the exact value of the area of the shaded region bounded by the curves $y = e^{2x} - 1$ and $y = 4e^{-2x} + 2$ and the y-axis.

(5)
(Total 15 marks)