Polynomials
Factor & Remainder Theorem
Question Paper 1

Level	International A Level
Subject	Maths
Exam Board	CIE
Topic	Algebra
Sub Topic	Polynomials – Factor & Remainder Theorem
Booklet	Question Paper 1

Time Allowed: **59 minutes**

Score: **/49**

Percentage: **/100**

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

1 The polynomials $f(x)$ and $g(x)$ are defined by

$$f(x) = x^3 + ax^2 + b \quad \text{and} \quad g(x) = x^3 + bx^2 - a,$$

where a and b are constants. It is given that $(x + 2)$ is a factor of $f(x)$. It is also given that, when $g(x)$ is divided by $(x + 1)$, the remainder is -18.

(i) Find the values of a and b. [5]

(ii) When a and b have these values, find the greatest possible value of $g(x) - f(x)$ as x varies. [2]

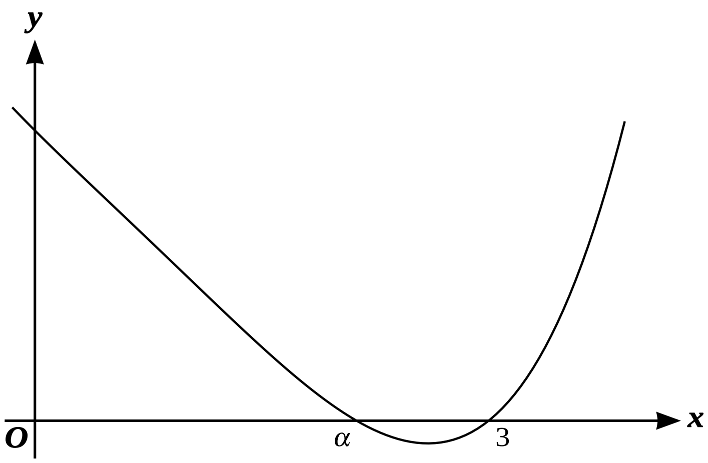
2 **(i)** Given that $(x + 2)$ is a factor of

$$4x^3 + ax^2 - (a + 1)x - 18,$$

find the value of the constant a. [3]

(ii) When a has this value, factorise $4x^3 + ax^2 - (a + 1)x - 18$ completely. [3]

3



The polynomial $p(x)$ is defined by

$$p(x) = x^4 - 3x^3 + 3x^2 - 25x + 48.$$

The diagram shows the curve $y = p(x)$ which crosses the x-axis at $(\alpha, 0)$ and $(3, 0)$.

(i) Divide $p(x)$ by a suitable linear factor and hence show that α is a root of the equation $x = \sqrt[3]{(16 - 3x)}$. [5]

(ii) Use the iterative formula $x_{n+1} = \sqrt[3]{(16 - 3x_n)}$ to find α correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

4 **(i)** Find the quotient when $6x^4 - x^3 - 26x^2 + 4x + 15$ is divided by $(x^2 - 4)$, and confirm that the remainder is 7. [3]

(ii) Hence solve the equation $6x^4 - x^3 - 26x^2 + 4x + 8 = 0$. [3]

5 The polynomial $\mathrm{p}(x)$ is defined by

$$\mathrm{p}(x) = x^3 + 2x + a,$$

where a is a constant.

(i) Given that $(x + 2)$ is a factor of $\mathrm{p}(x)$, find the value of a. [2]

(ii) When a has this value, find the quotient when $\mathrm{p}(x)$ is divided by $(x + 2)$ and hence show that the equation $\mathrm{p}(x) = 0$ has exactly one real root. [5]

6 **(i)** The polynomial $x^3 + ax^2 + bx + 8$, where a and b are constants, is denoted by $\mathrm{p}(x)$. It is given that when $\mathrm{p}(x)$ is divided by $(x - 3)$ the remainder is 14, and that when $\mathrm{p}(x)$ is divided by $(x + 2)$ the remainder is 24. Find the values of a and b. [5]

(ii) When a and b have these values, find the quotient when $\mathrm{p}(x)$ is divided by $x^2 + 2x - 8$ and hence solve the equation $\mathrm{p}(x) = 0$. [4]

7 Solve the equation $2\cot^2\theta - 5\,\mathrm{cosec}\,\theta = 10$, giving all solutions in the interval $0° \leqslant \theta \leqslant 360°$. [6]